Generalized action principle and extrinsic geometry for N=1 superparticle.

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Abstract

It is proposed the generalized action functional for N=1 superparticle in D=3,4,6 and 10 space – time dimensions. The superfield geometric approach equations describing superparticle motion in terms of extrinsic geometry of the worldline superspace are obtained on the base of the generalized action. The off – shell superdiffeomorphism invariance (in the rheonomic sense) of the superparticle generalized action is proved. It was demonstrated that the half of the fermionic and one bosonic (super)fields disappear from the generalized action in the analytical basis.

Superparticle interaction with Abelian gauge theory is considered in the framework of this formulation. The geometric approach equations describing superparticle motion in Abelian background are obtained.

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1 Introduction.

Recently the generalized action functional for superstrings and super – p – branes was proposed [1]. It solved some problems of the standard twistor – like (doubly supersymmetric or world – volume superfield) formulations [2] – [18] (see also Refs. in [19]) related to the presence in the latter ones of the Lagrange multiplier superfields, which lead to the appearance of additional (undesirable) propagating degrees of freedom [15, 19] and of the infinitely reducible symmetries [6, 18] in the formulations of type IIB D=10 superstrings and D=11 supermembranes.

So, there is a hope that the generalized action principle can help to solve the problem of completely covariant quantization of D=10 superstring [2].

At the same time the generalized action can be used to reproduce naturally [31, 32] the doubly supersymmetric generalization [19] of the geometric approach [33], which can be useful for the more deep understanding of the group – theoretical and geometrical properties of (super)string and (super)membrane theory [31] – [36] and, in particular, for investigation of the problems of coupling supersymmetric extended objects with natural (supergravity) background.

Up to now the generalized action principle together with doubly supersymmetric geometrical approach was not developed for the case of superparticles. However, they can provide, in particular, toy models for looking for a way of covariant quantization in the framework of the generalized action principle approach [3] and for studying the general properties of geometrical approach to supersymmetric objects in flat and curved space – time. Moreover, the case of (super)particle has some peculiar features, which are absent for superstring and super – p – branes, but have to be in tensionless supersymmetric extended objects: the so – called null – superstrings and null – super – p – branes [37, 24]. These objects become very attractive in relation to the string cosmology problems [38, 39]. Besides, they may be related to the M – theory compactifications to six dimensions [40].

So, this paper can be regarded as ABC of the generalized action principle and geometric approach to supersymmetric objects (interacting with background fields). We construct the generalized action functional for N=1 superparticle in D=3,4,6,10, investigate its

[2]Recently *Lorentz* ($SO(1,3)$) covariant quantization of heterotic string compactified on Calabi – Yau manifold has been carried out in [20] using conformal field theory approach. Most successful previous attempts to reach a progress in Lorentz ($SO(1,D-1)$) covariant quantization of D=10 superstring are ones presented in Refs. [21, 22]. All principal problems on this way were solved in the frame of the so – called twistor – like Lorentz – harmonic formulation [23] (see also [24], [25], [26]), however the calculation problems [23] hamper a progress on this line. Partially covariant quantization was studied in [27, 28, 29, 30]. For recent attempts to find alternative way see [54]

[3]The study of the covariant quantization problem is not so simple subject due to non – linear realization of some symmetries in the considered formulation. This is the main obstacle for straightforward application of known modern quantization schemes and incites in a search of new versions of covariant quantization to overcome this problem.

properties and symmetries, derive equations of motion in terms of worldline superfield and construct the geometric approach for superparticle.

On shell, the geometric approach equations describe free superparticle motion and give not any new information. However, as it was demonstrated in our paper, when we omit proper dynamical equation from the consideration, the rest of the geometric approach equations are appropriate for description of superparticle motion in any (Abelian) gauge field.

As preliminary steps for looking for a way for covariant quantization in the frame of the generalized action approach, we demonstrate that the generalized action for superparticle satisfies the conditions of the off – shell superdiffeomorphism invariance (in rheonomic sense [41]) and prove that in the analytical coordinate basis Sokatchev's effect of disappearance of one bosonic and half of the fermionic (super)fields (being the compensators for nonlinearly realized reparametrization and worldline supersymmetry) [42, 43] takes place in the generalized action functional.

An investigation of superparticle interaction with Abelian gauge (super)field in the generalized action principle approach demonstrates that coupling leads to the restriction of background field strength by the standard set of superspace constraints (compare with [44] and Refs. therein). We derive the geometric approach equations for N=1 superparticle interacting with Abelian gauge background and find the correspondence of the superfields describing the extrinsic geometry of the worldline superspace with worldline image of the gauge field strength superfield.

Notations: the underlined indices belong to a target superspace, $(++,--)$ and $(+,-)$ denote $SO(1,1)$ weights, $(q,\dot{q},i)$ are the indices of spinor and vector representations of $SO(D-2)$ (for D=10 q and $\dot{q}$ are the **s** and **c** spinor indices of $SO(8)$).

2 N=1 superparticle in diverse dimensions.

2.1 The generalized action for superparticle and moving frame variables.

The generalized action describing superparticle in D=3,4,6,10 dimensions has the following form

$$S=\int_{\mathcal{M}^1}\rho^{++}E^{--}\equiv\int_{\mathcal{M}^1}\rho^{++}(dX^{\underline{m}}-id\Theta\Gamma^{\underline{m}}\Theta)u_{\underline{m}}^{--} \tag{1}$$

The Lagrangian one – form

$$\mathcal{L}=\rho^{++}E^{--}$$

is integrated over one – dimensional bosonic submanifold

$$\mathcal{M}^1=\{(\tau,\eta^q(\tau))\}$$

of worldline superspace

$$\Sigma^{(1|D-2)} = \{(\tau, \eta^q)\}$$

parametrized by the proper – time τ and its Grassmann superpartner η^q ($q = 1, \ldots, D-2$). Thus, the coordinates of a target superspace of the model, including D – dimensional space – time coordinates $X^{\underline{m}}$ ($m = 0, 1, \ldots, D-1$), their superpartners $\Theta^{\underline{\mu}}$ ($\underline{\mu} = 1, \ldots, 2(D-2)$) as well as scalar density ρ^{++} and light – like moving frame vector $u^{--}_{\underline{m}}$ (see below) shall be regarded in (1) as worldline superfields

$$X^{\underline{m}} = X^{\underline{m}}(\tau, \eta^q), \quad \Theta^{\underline{\mu}} = \Theta^{\underline{\mu}}(\tau, \eta^q), \quad \rho^{++} = \rho^{++}(\tau, \eta^q), \quad u^{--}_{\underline{m}} = u^{--}_{\underline{m}}(\tau, \eta^q)$$

but taken on (an arbitrary) bosonic line $\mathcal{M}^1$ ($\eta^q = \eta^q(\tau)$):

$$X^{\underline{m}} = X^{\underline{m}}(\tau, \eta^q(\tau)), \quad \Theta^{\underline{\mu}} = \Theta^{\underline{\mu}}(\tau, \eta^q(\tau)), \quad \rho^{++} = \rho^{++}(\tau, \eta^q(\tau)), \quad u^{--}_{\underline{m}} = u^{--}_{\underline{m}}(\tau, \eta^q(\tau))$$

The number of η^q is considered to be half of the number of spinor target superspace variables $\Theta^{\underline{\mu}}$, whose number coincides with the dimension of the minimal spinor representation, being $2(D-2)$ ($\underline{\mu} = 1, \ldots, 2(D-2)$) for the considered cases D=3,4,6,10.

In distinction to the superstring and supermembrane cases [1], none of the components of the worldline supereinbein superfields $e^A_M(\tau, \eta^q)$

$$e^A \equiv (e^{++}, e^{+p}) = d\tau e^A_\tau(\tau, \eta^q) + d\eta^q e^A_q(\tau, \eta^q)$$

are involved into the action functional (1). However, they appear in the decomposition of the external differential

$$d = e^{++} D_{++} + e^{+q} D_{+q},$$

where D_{+q} and D_{++} are the covariant derivatives for worldline scalar superfields.

In (1) ρ^{++} is an arbitrary scalar density superfield and

$$E^{--} = \Pi^{\underline{m}} u^{--}_{\underline{m}} = (dX^{\underline{m}} - i d\Theta \Gamma^{\underline{m}} \Theta) u^{--}_{\underline{m}}$$

is an appropriate part of arbitrary local (co) – frame $E^{\underline{A}} = (E^{\underline{a}}, E^{\underline{\alpha}})$ of flat target superspace. Supervielbein $E^{\underline{A}}$ is related to the basic supercovariant forms [46]

$$\Pi^{\underline{m}} = dX^{\underline{m}} - i d\Theta \Gamma^{\underline{m}} \Theta, \qquad d\Theta^{\underline{\mu}} \tag{2}$$

by $SO(1, D-1)$ rotations

$$E^{\underline{A}} \equiv (E^{\underline{a}}, E^{\underline{\alpha}}) = (\Pi^{\underline{m}}, d\Theta^{\underline{\mu}}) \begin{pmatrix} u^{\underline{a}}_{\underline{m}} & 0 \\ 0 & v^{\underline{\alpha}}_{\underline{\mu}} \end{pmatrix}$$

$$E^{\underline{a}} \equiv (E^{++}, E^{--}, E^i) = \Pi^{\underline{m}} u^{\underline{a}}_{\underline{m}} \equiv (\Pi^{\underline{m}} u^{++}_{\underline{m}}, \Pi^{\underline{m}} u^{--}_{\underline{m}}, \Pi^{\underline{m}} u^i_{\underline{m}})$$

$$E^{\underline{\alpha}} \equiv (E^{+q}, E^{-\dot{q}}) = d\Theta^{\underline{\mu}} v^{\underline{\alpha}}_{\underline{\mu}} \equiv (d\Theta^{\underline{\mu}} v^+_{\underline{\mu} q}, d\Theta^{\underline{\mu}} v^-_{\underline{\mu}\dot{q}}) \tag{3}$$

which vector and spinor representations are given by matrices $u_{\underline{m}}^{\underline{a}} = (u_{\underline{m}}^{++}, u_{\underline{m}}^{--}, u_{\underline{m}}^{i})$ and $v_{\underline{\mu}}^{\underline{\alpha}} = (v_{\underline{\mu}q}^{+}, v_{\underline{\mu}\dot{q}}^{-})$ respectively

$$u_{\underline{m}}^{\underline{a}} = (u_{\underline{m}}^{\pm\pm}, u_{\underline{m}}^{i}) \in SO(1, D-1) \Longleftrightarrow u_{\underline{m}}^{\underline{a}} u^{\underline{mb}} = \eta^{\underline{ab}} \tag{4}$$

$$v_{\underline{\mu}}^{\underline{\alpha}} = (v_{\underline{\mu}q}^{+}, v_{\underline{\mu}\dot{q}}^{-}) \in Spin(1, D-1) \tag{5}$$

The components $u_{\underline{m}}^{\pm\pm}, u_{\underline{m}}^{i}$, $v_{\underline{\mu}q}^{+}, v_{\underline{\mu}\dot{q}}^{-}$ of these matrices can be treated as the vector and spinor Lorentz harmonic variables [42, 43], [24] – [26]. Due to the $SO(1,1) \otimes SO(D-2)$ gauge symmetry of the considered action (1) (see below) they can be regarded as coordinates of non – compact coset space $SO(1, D-1)/SO(1,1) \otimes SO(D-2)$ (see Refs. [42, 43], [23]–[26], [19] for details).

Specific point of superparticle theory is the presence of K_{D-2} gauge symmetry in the action (1) (see below). As a result, regarding the gauge symmetry $SO(1,1) \otimes SO(D-2) \otimes K_{D-2}$ as an identification relation on the space of harmonic (moving frame) variables, one can identifies them with coordinates of the compact coset space $S^{D-2} = SO(1, D-1)/(SO(1,1) \otimes SO(D-2) \otimes K_{D-2})$ [48], which can be regarded as a celestial sphere [49].

Being the vector and spinor representation of the same $SO(1, D-1)$ rotation, u and v variables are related by the condition of the conservation of D – dimensional gamma – matrices

$$u_{\underline{m}}^{\underline{a}} (\Gamma^{\underline{m}})_{\underline{\mu\nu}} = v_{\underline{\mu}}^{\underline{\alpha}} (\Gamma^{\underline{a}})_{\underline{\alpha\beta}} v_{\underline{\nu}}^{\underline{\beta}} \tag{6}$$

Eq. (6) is a generalization [48, 23, 26] of Cartan – Penrose representation [49, 50] of light – like vector (see Eqs. (30), (31), (50), (51) below).

Eq. (6) can be regarded as a manifestation of the composed nature of vector harmonic variables. From the other hand u and v are the representations of the same rotations and, thus, we have a possibility to consider our system in both terms of u variables or v ones whenever it is useful.

In the framework of geometrical approach [19, 33] $u_{\underline{a}}^{\underline{m}}$ should be considered as a local moving frame attached to each point of worldvolume supersurface and $v_{\underline{\alpha}}^{\underline{\mu}}$ should be regarded [23, 26] as a generalization of Newman – Penrose dyades [47]. Then an infinitesimal transport shall change the moving frame variables by Lorentz rotation parametrized by $so(1, D-1)$ valued Cartan forms $\Omega^{\underline{ab}} = -\Omega^{\underline{ba}}$

$$du_{\underline{m}}^{\underline{b}} = u_{\underline{m}}{}^{\underline{a}} \Omega_{\underline{a}}{}^{\underline{b}}; \qquad dv_{\underline{\mu}}^{\underline{\alpha}} = \frac{1}{4} \Omega^{\underline{ab}} v_{\underline{\mu}}^{\underline{\beta}} (\Gamma_{\underline{ab}})_{\underline{\beta}}{}^{\underline{\alpha}}; \qquad \Omega^{\underline{ab}} = u_{\underline{m}}^{\underline{a}} du^{\underline{bm}} \tag{7}$$

With taking into account Eqs. (3) and (6),

$$u_{\underline{m}}^{\underline{a}} = (u_{\underline{m}}^{\pm\pm}, u_{\underline{m}}^{i}), \quad v_{\underline{\mu}}{}^{\underline{\alpha}} = (v_{\underline{\mu}q}^{+}, v_{\underline{\mu}\dot{q}}^{-}), \quad v_{\underline{\alpha}}{}^{\underline{\mu}} = (v_{q}^{-\underline{\mu}}, v_{\dot{q}}^{+\underline{\mu}}), \quad v_{\underline{\alpha}}^{\underline{\mu}} v_{\underline{\mu}}^{\underline{\beta}} = \delta_{\underline{\alpha}}^{\underline{\beta}}$$

it is naturally to split Cartan form $\Omega^{\underline{ab}}$ (7) into the set of:

- $SO(1,1)$ connection

$$\Omega^{(0)}(d) \equiv \frac{1}{2}u_{\underline{m}}^{--}du^{++\underline{m}} = -\frac{2}{D-2}v_{\dot{q}}^{+\underline{\mu}}dv_{\underline{\mu}\dot{q}}^{-} = \frac{2}{D-2}v_{q}^{-\underline{\mu}}dv_{\underline{\mu}q}^{+} \tag{8}$$

- $SO(D-2)$ connection

$$\Omega^{ij}(d) \equiv u_{\underline{m}}^{i}du^{j\underline{m}} = \frac{2}{D-2}v_{q}^{-\underline{\mu}}\gamma_{qp}^{ij}dv_{\underline{\mu}p}^{+} = \frac{2}{D-2}v_{\dot{q}}^{+\underline{\mu}}\tilde{\gamma}_{\dot{q}\dot{p}}^{ij}dv_{\underline{\mu}\dot{p}}^{-} \tag{9}$$

- $SO(1,D-1)/(SO(1,1)\otimes SO(D-2))$ vielbeins

$$\Omega^{++i} \equiv u_{\underline{m}}^{++}du^{i\underline{m}} = \frac{2}{D-2}v_{\dot{q}}^{+\underline{\mu}}\tilde{\gamma}_{\dot{q}q}^{i}dv_{\underline{\mu}q}^{+} \tag{10}$$

$$\Omega^{--i} \equiv u_{\underline{m}}^{--}du^{i\underline{m}} = \frac{2}{D-2}v_{q}^{-\underline{\mu}}\gamma_{q\dot{q}}^{i}dv_{\underline{\mu}\dot{q}}^{-} \tag{11}$$

By definition $\Omega^{\underline{ab}}$ satisfies the Maurer – Cartan equations

$$d\Omega^{\underline{ab}} - \Omega^{\underline{a}}{}_{\underline{c}}\Omega^{\underline{cb}} = 0 \tag{12}$$

(which are the integrability conditions for (7)). Having in mind the $SO(1,1)\otimes SO(D-2)$ invariant decomposition (8) – (11) we can split (12) into the following set of equations:

$$\mathcal{D}\Omega^{++i} \equiv d\Omega^{++i} - \Omega^{++i}\Omega^{(0)} + \Omega^{++j}\Omega^{ji} = 0 \tag{13}$$

$$\mathcal{D}\Omega^{--i} \equiv d\Omega^{--i} + \Omega^{--i}\Omega^{(0)} + \Omega^{--j}\Omega^{ji} = 0 \tag{14}$$

$$\mathcal{F} \equiv d\Omega^{(0)} = \frac{1}{2}\Omega^{--i}\Omega^{++i} \tag{15}$$

$$R^{ij} \equiv d\Omega^{ij} + \Omega^{ik}\Omega^{kj} = -\Omega^{--[i}\Omega^{++j]} \tag{16}$$

In the case of bosonic p – branes with $p \geq 1$ Eqs. (13) – (16) give rise to Peterson – Codazzi, Gauss and Ricci equations of surface theory respectively (see [33] and Refs. therein).

2.2 Symmetries and equations of motion.

To investigate the gauge symmetries of the action (1) and to obtain the equations of motion, let us consider the variation of the functional (1) with respect to the superfield variables (modulo boundary terms):

$$\delta S = \int_{\mathcal{M}^1} \delta\rho^{++}E^{--} + \int_{\mathcal{M}^1} \rho^{++}(d\Pi^{\underline{m}}(\delta)u_{\underline{m}}^{--} - 2id\Theta\gamma^{\underline{m}}\delta\Theta u_{\underline{m}}^{--} + \Pi^{\underline{m}}\delta u_{\underline{m}}^{--}) \tag{17}$$

$$= \int_{\mathcal{M}^1} (\delta\rho^{++} - \rho^{++}\Omega^{(0)}(\delta))E^{--} + \int_{\mathcal{M}^1} \rho^{++}\Omega^{--i}(\delta)E^{i}$$

$$- \int_{\mathcal{M}^1} \rho^{++}E^{i}(\delta)\Omega^{--i}(d) - 4i\int_{\mathcal{M}^1} \rho^{++}E^{-\dot{q}}E^{-\dot{q}}(\delta)$$

$$- \int_{\mathcal{M}^1} E^{--}(\delta) D\rho^{++} \tag{18}$$

To get manifestly supersymmetric expression, the supersymmetric invariant variation

$$\Pi^{\underline{m}}(\delta) = \delta X^{\underline{m}} - i\delta\Theta\gamma^{\underline{m}}\Theta$$

is used in (17) instead of $\delta X^{\underline{m}}$:

$$E^{--}(\delta) = \Pi^{\underline{m}}(\delta) u_{\underline{m}}^{--}; \qquad E^{i}(\delta) = \Pi^{\underline{m}}(\delta) u_{\underline{m}}^{i}; \qquad E^{-\dot{q}}(\delta) \equiv \delta\Theta^{\underline{\mu}} v_{\underline{\mu}}^{-\dot{q}} \tag{19}$$

In accordance with (7) variations of moving frame variables are described by Cartan forms (8) – (11) dependent on the variation symbol δ instead of d. Thus, $\Omega^{(0)}(\delta)$ and $\Omega^{--i}(\delta)$ are the parameters of $SO(1,1)$ and $S^{D-2} \simeq SO(1,D-1)/(SO(1,1) \otimes SO(D-2) \otimes K_{D-2})$ [48] transformations respectively. The absence of the parameters $\Omega^{ij}(\delta)$, $\Omega^{++i}(\delta)$ of $SO(D-2)$ and K_{D-2} transformations reflects the gauge symmetries of the theory (see below).

In (17), (18) and below the covariant derivative

$$D\rho^{++} = d\rho^{++} - \rho^{++}\Omega^{(0)}(d)$$

$$DE^{--}(\delta) \equiv dE^{--}(\delta) + E^{--}(\delta)\Omega^{(0)}(d)$$

$$DE^{i} = dE^{i} + E^{j}\Omega^{ij}, \quad \ldots$$

is defined by spin ($SO(1,1)$) connection $\Omega^{(0)}(d)$ and $SO(D-2)$ gauge fields $\Omega^{ij}(d)$ (i.e. by pullback of (8), (9)) being induced by embedding.

From (18) it is easy to see that, in addition to N=1 target space supersymmetry, the action (1) possesses the following symmetries [4]:

1) $SO(1,1)$ symmetry

$$\Omega^{(0)}(\delta) = 2\alpha; \quad \delta\rho^{++} = 2\alpha\rho^{++}; \quad \delta u_{\underline{m}}^{++} = 2\alpha u_{\underline{m}}^{++}; \quad \delta u_{\underline{m}}^{--} = -2\alpha u_{\underline{m}}^{--} \tag{20}$$

$$\delta v_{\underline{\mu}q}^{+} = \alpha v_{\underline{\mu}q}^{+}; \qquad \delta v_{\underline{\mu}\dot{q}}^{-} = -\alpha v_{\underline{\mu}\dot{q}}^{-}$$

2) The symmetry under $SO(D-2)$ rotations

$$\Omega^{ij}(\delta) = \alpha^{ij}; \quad \delta u_{\underline{m}}^{i} = -u_{\underline{m}}^{j}\alpha^{ji}; \quad \delta v_{\underline{\mu}q}^{+} = -\frac{1}{4}\alpha^{ij} v_{\underline{\mu}p}^{+}\gamma_{pq}^{ij}; \quad \delta v_{\underline{\mu}\dot{q}}^{-} = -\frac{1}{4}\alpha^{ij} v_{\underline{\mu}\dot{p}}^{-}\tilde{\gamma}_{\dot{p}\dot{q}}^{ij}$$

reflecting the absence of $u_{\underline{m}}^{i}$ variables in the action (1) (but $u_{\underline{m}}^{i}$ are present indirectly due to the orthogonality constraint (4) on $u_{\underline{m}}^{++}$ variables $u_{\underline{m}}^{++}u^{i\underline{m}} = 0$). It manifests itself in absence of $\Omega^{ij}(\delta)$ variation in (18).

[4] For the reader convenience, we present below the transformations of both u and v variables, though they are completely determined by the expressions for Cartan forms $\Omega^{\underline{ab}}(\delta)$ in accordance with Eqs. (7), (8) – (11).

3) K_{D-2} "boost" symmetry [48, 24] arising from the absence of $v^+_{\underline{\mu}q}$ (or $u^{++}_{\underline{m}}$, $u^i_{\underline{m}}$) variables in the action (1)

$$\Omega^{++i}(\delta) = k^{++i}; \quad \delta u^{++}_{\underline{m}} = u^i_{\underline{m}} k^{++i}; \quad \delta u^i_{\underline{m}} = \frac{1}{2} u^{--}_{\underline{m}} k^{++i}; \quad \delta v^+_{\underline{\mu}q} = \frac{1}{2} k^{++i} \gamma^i_{q\dot{q}} v^-_{\underline{\mu}\dot{q}} \quad (21)$$

4) $n = D-2$ local worldline fermionic symmetry realized nonlinearly as superfield irreducible κ – symmetry [45, 7]

$$\delta\Theta^{\underline{\mu}} v^+_{\underline{\mu}q} = \kappa^{+q}; \qquad \delta X^{\underline{m}} = i\delta\Theta\gamma^{\underline{m}}\Theta \quad (22)$$

5) Bosonic superfield b – symmetry [45, 7, 18] (related to reparametrization symmetry)

$$\delta X^{\underline{m}} = E^{++}(\delta) u^{--\underline{m}} \equiv b^{++}(\tau, \eta^{+q}) u^{--\underline{m}}, \qquad \delta\Theta^{\underline{\mu}} = 0. \quad (23)$$

which close the algebra of the κ – symmetry transformations.

All the symmetries of the proposed action (1) are realized in completely irreducible way [5] (in contrast to the previously known superfield formulations, especially in the case of D=10 superparticle [6]). Some of them will be used below for establishing a connection to the usual superfield versions [2] – [18].

Equations of motion [6] derived by variation over $\delta\rho^{++}$, $\Omega^{--i}(\delta)$, $E^{--}(\delta)$, $E^{-\dot{q}}(\delta) = \delta\Theta^{\underline{\mu}} v^-_{\underline{\mu}\dot{q}}$ and $E^i(\delta)$ have the following form:

$$E^{--} = 0, \qquad \rho^{++} E^i = 0 \quad (24)$$

$$\rho^{++} E^{-\dot{q}} \equiv \rho^{++} d\Theta^{\underline{\mu}} v^{-\dot{q}}_{\underline{\mu}} = 0 \quad (25)$$

$$\Omega^{--i}(d) = 0 \quad (26)$$

$$D\rho^{++} \equiv d\rho^{++} - \rho^{++}\Omega^{(0)}(d) = 0 \quad (27)$$

The variation with respect to the surface $\mathcal{M}^1$ ($\frac{\delta S}{\delta\eta^q(\tau)} = 0$) does not result in independent equation of motion. This reflects superdiffeomorphism invariance of the generalized action [1] (see also subsection 2.4) and provides the possibility to treat Eqs. (24) – (27) as superfield equations, which hold on the whole worldline superspace $\Sigma^{(1|D-2)}$ [1].

Supposing $\rho^{++} \neq 0$ we can solve (24) as

$$\Pi^{\underline{m}} \equiv dX^{\underline{m}} - id\Theta\Gamma^{\underline{m}}\Theta = \frac{1}{2} e^{++} u^{--\underline{m}} \quad (28)$$

[5]Pure component counterparts of all the considered symmetries are present in Lorentz – harmonic formulation of superparticles and null – super – p – branes [24].

[6]We would like to point out the fact that equations of motion derived from the action can be splitted into the two different parts: proper (dynamical) equations of motion and "rheotropic relations" [1] establishing a connection between target – space and worldline vielbeins.

and identify by this a pullback of the target – space one form $E^{++} = \Pi^{\underline{m}} u^{++}_{\underline{m}}$ with worldline "einbein" e^{++}

$$E^{++} = e^{++} \tag{29}$$

This choice is possible due to the absence of worldline supereinbein one – form in the original action.

It shall be stressed, that due to (6)

$$u^{--}_{\underline{m}} \delta_{\dot{p}\dot{q}} = v^{-}_{\underline{\mu}\dot{q}} \Gamma_{\underline{m}} v^{-}_{\underline{\mu}\dot{p}} \Longrightarrow u^{--}_{\underline{m}} = \frac{1}{D-2} v^{-}_{\underline{\mu}\dot{q}} \Gamma_{\underline{m}} v^{-}_{\underline{\mu}\dot{p}} \tag{30}$$

and, hence (28) reproduces the twistor – like solution

$$\Pi^{\underline{m}}_{++} \equiv D_{++} X^{\underline{m}} - i D_{++} \Theta \Gamma^{\underline{m}} \Theta = \frac{1}{D-2} v^{-}_{\underline{\mu}\dot{q}} \Gamma_{\underline{m}} v^{-}_{\underline{\mu}\dot{p}} \tag{31}$$

of the mass shell condition

$$\Pi^{\underline{m}}_{++} \Pi_{++\underline{m}} = 0$$

Moreover, the fact that $\Pi^{\underline{m}}$ is expressed in terms of only vector vielbein e^{++}, while $d = e^{++} D_{++} + e^{+q} D_{+q}$, means that we have obtained the "geometrodynamical" condition [6], [15]

$$\Pi^{\underline{m}}_{+q} \equiv D_{+q} X^{\underline{m}} - i D_{+q} \Theta \Gamma^{\underline{m}} \Theta = 0 \tag{32}$$

being the starting point of the standard doubly supersymmetric approach [2, 4, 6, 15], as equation of motion without any use of Lagrange multipliers.

From equation (25) for $\Theta^{\underline{\mu}}$ variables

$$\rho^{++} d\Theta^{\underline{\mu}} v^{-}_{\underline{\mu}\dot{q}} = 0 \tag{33}$$

we get rheotropic relation [1]

$$D_{+q} \Theta^{\underline{\mu}} v^{-}_{\underline{\mu}\dot{q}} = 0 \tag{34}$$

as well as the proper equation of motion

$$D_{++} \Theta^{\underline{\mu}} v^{-}_{\underline{\mu}\dot{q}} = 0 \tag{35}$$

Below we will prove that (35) is the only independent dynamical superfield equation of superparticle theory, because Eq. (26) results from (35).

Eq. (34) means that

$$D_{+q} \Theta^{\underline{\mu}} = A_q{}^p v_p^{-\underline{\mu}}$$

with an arbitrary matrix $A_q{}^p$ supposed to be non – degenerate (i.e. $det\ A \neq 0$).

Since e^{+q} variables are not involved into construction of our action, we can choose it to be induced by embedding, i.e. we can suppose

$$E^{+q} = d\Theta^{\underline{\mu}} v^{+}_{\underline{\mu}q},$$

as we have done it for e^{++} in (29). To make this point clear, let us note that, due to the absence of e^{+q} in the action (1), we have a symmetry [1]

$$e^{+q} \mapsto \tilde{e}^{+q} = (e^{+p} + e^{++}\chi^{+p}_{++}) W_p{}^q, \qquad detW \neq 0 \tag{36}$$

$$\Longrightarrow D_{+q} \mapsto \tilde{D}_{+p} W_q^{-1p}; \qquad D_{++} \mapsto \tilde{D}_{++} - \chi^{+p}_{++}\tilde{D}_{+p}$$

which allows us to obtain the relation $A_q{}^p = \delta_q{}^p$, or

$$D_{+q}\Theta^{\underline{\mu}} = v^{\underline{\mu}}_{+q} \tag{37}$$

as well as

$$D_{++}\Theta^{\underline{\mu}} v^{+}_{\underline{\mu}q} = 0. \tag{38}$$

So Eq. (37) identifies the generalized twistor superfield $D_{+q}\Theta^{\underline{\mu}}$ [2] with spinor harmonic $v_q^{-\underline{\mu}}$ [24, 48], [23] – [26] (see [19] for super – p – brane case).

Eqs. (37), (38) can be represented by the one – form equations

$$E^{+q} \equiv d\Theta^{\underline{\mu}} v^{+q}_{\underline{\mu}} = e^{+q}, \qquad E^{-\dot{q}} \equiv d\Theta^{\underline{\mu}} v^{-}_{\underline{\mu}\dot{q}} = e^{++} D_{++}\Theta^{\underline{\mu}} v^{-}_{\underline{\mu}\dot{q}} \equiv e^{++}\psi^{-}_{++\dot{q}} \tag{39}$$

Henceforth, a pullback of target – space Grassmann one – form looks like

$$d\Theta^{\underline{\mu}} = e^{+q} v_q^{-\underline{\mu}} + e^{++}\psi^{-}_{++\dot{q}} v_{\dot{q}}^{+\underline{\mu}} \tag{40}$$

and equation of motion (35) means

$$\psi^{-}_{++\dot{q}} = 0$$

.

Thus, we have obtained the set of rheotropic conditions (24), (28), (40) and the proper equations of motion (27), (26) and (35).

All these relations are written in terms of differential forms, so it is easy to investigate their integrability conditions.

2.3 "Off – shell" description of N=1 superparticle.

Let us, for a time, omit from the consideration the proper dynamical equations of motion (35) (and (26)) as well as Eq. (27) for Lagrange multiplier superfield ρ^{++} and study the integrability condition for rheotropic relations (24), (37), (29) written in the form (28), (40). They are [7]

$$d\Pi^{\underline{m}} \equiv -id\Theta\Gamma^{\underline{m}} d\Theta = \frac{1}{2} e^{++} D u^{--\underline{m}} + \frac{1}{2} D e^{++} u^{--\underline{m}}$$

[7] Remind that we use external differential and external product of forms, so $dd = 0$ and $\Omega_S\Omega_R = (-)^{SR}\Omega_R\Omega_S$, $d(\Omega_R\Omega_S) = \Omega_R d\Omega_S + (-)^S d\Omega_R\Omega_S$ for products of any bosonic R – and S – forms $\Omega_R = dZ^{\underline{M}_R} \ldots dZ^{\underline{M}_1}\Omega_{\underline{M}_1 \ldots \underline{M}_R}$ and $\Omega_S = dZ^{\underline{M}_S} \ldots dZ^{\underline{M}_1}\Omega_{\underline{M}_1 \ldots \underline{M}_S}$.

$$= \frac{1}{2} e^{++} \Omega^{--i} u^{i\underline{m}} + \frac{1}{2} T^{++} u^{--\underline{m}} \tag{41}$$

$$0 \equiv dd\Theta^{\underline{\mu}} = De^{+q} v_q^{-\underline{\mu}} + De^{++} \psi^-_{++\dot{q}} v_{\dot{q}}^{+\underline{\mu}} + e^{++} D\psi^-_{++\dot{q}} v_{\dot{q}}^{+\underline{\mu}} + e^{+q} Dv_q^{-\underline{\mu}} + e^{++} \psi^-_{++\dot{q}} Dv_{\dot{q}}^{+\underline{\mu}}$$

$$= (T^{+q} - \frac{1}{2} e^{++} \wedge \Omega^{++i} \gamma^i_{q\dot{q}} \psi^-_{++\dot{q}}) v_q^{-\underline{\mu}} + (T^{++} \psi^-_{++\dot{q}} + e^{++} D\psi^-_{++\dot{q}} - \frac{1}{2} e^{+q} \Omega^{--i} \gamma^i_{q\dot{q}}) v_{\dot{q}}^{+\underline{\mu}} \tag{42}$$

where the covariant differential D involves $SO(1,1)$ and $SO(D-2)$ connections induced by embedding, i.e. coincident with the pullbacks of the forms $\Omega^{(0)}(d)$ and $\Omega^{ij}(d)$. The later fact can be represented by the following relations

$$\Omega^{(0)}(D) = 0; \qquad \Omega^{ij}(D) = 0$$

As a result, covariant differentials of harmonic variables involve the $SO(1,D-1)/SO(1,1)\otimes SO(D-2)$ forms only (compare with (7)). To make this point clear let us write the explicit expressions for D=10 case, where $\gamma^i_{q\dot{q}}$ is the $SO(8)$ γ – matrices, $\tilde{\gamma}^i_{\dot{q}q} = \gamma^i_{q\dot{q}}$, $\gamma^{ij} = \gamma^{[i}\tilde{\gamma}^{j]}$, $\tilde{\gamma}^{ij} = \tilde{\gamma}^{[i}\gamma^{j]}$:

$$Dv_q^{-\underline{\mu}} \equiv dv_q^{-\underline{\mu}} + \frac{1}{2} \Omega^{(0)} v_q^{-\underline{\mu}} - \frac{1}{4} \Omega^{ij} \gamma^{ij}_{qp} v_p^{-\underline{\mu}} = -\frac{1}{2} \Omega^{--i} \gamma^i_{q\dot{q}} v_{\dot{q}}^{+\underline{\mu}}$$

$$Dv_{\dot{q}}^{+\underline{\mu}} \equiv dv_{\dot{q}}^{+\underline{\mu}} - \frac{1}{2} \Omega^{(0)} v_{\dot{q}}^{+\underline{\mu}} - \frac{1}{4} \Omega^{ij} \tilde{\gamma}^{ij}_{\dot{q}\dot{p}} v_{\dot{p}}^{+\underline{\mu}} = -\frac{1}{2} \Omega^{++i} \gamma^i_{q\dot{q}} v_q^{-\underline{\mu}}$$

$$Du^{--}_{\underline{m}} \equiv du^{--}_{\underline{m}} + \Omega^{(0)} u^{--}_{\underline{m}} = u^i_{\underline{m}} \Omega^{--i}, \quad \ldots \tag{43}$$

In Eqs. (41), (42) T^{++} and T^{+q} are the bosonic and fermionic torsion two – forms

$$T^{++} \equiv De^{++} = de^{++} - e^{++} \Omega^{(0)} \tag{44}$$

$$T^{+q} \equiv De^{+q} = de^{+q} - \frac{1}{2} e^{+q} \Omega^{(0)} + \frac{1}{4} e^{+p} \Omega^{ij} \gamma^{ij}_{pq} \tag{45}$$

Contracting (41) and (42) with moving frame vectors $\frac{1}{2} u^{++}_{\underline{m}}, u^i_{\underline{m}}$ and spinor harmonics $v^+_{\underline{\mu}q}, v^-_{\underline{\mu}\dot{q}}$ we get:

$$T^{++} = -2iE^{+q} E^{+q} \tag{46}$$

$$e^{++} \Omega^{--i} = -4iE^{+q} \gamma^i_{q\dot{q}} E^{-\dot{q}} \tag{47}$$

$$T^{+q} = \frac{1}{2} e^{++} \wedge \Omega^{++i} \gamma^i_{q\dot{q}} \psi^-_{++\dot{q}} \tag{48}$$

$$e^{+q} \Omega^{--i} \gamma^i_{q\dot{q}} = 2e^{++} D\psi^-_{++\dot{q}} + 2T^{++} \psi^-_{++\dot{q}} \tag{49}$$

where the consequences of Eq. (6)

$$u^{++}_{\underline{m}} \Gamma^{\underline{m}}_{\underline{\mu\nu}} = 2v^+_{\underline{\mu}q} v^+_{\underline{\nu}q} \tag{50}$$

$$u^i_{\underline{m}} \Gamma^{\underline{m}}_{\underline{\mu\nu}} = 2v^+_{\{\underline{\mu}|q} \gamma^i_{q\dot{q}} v^-_{|\underline{\nu}\}\dot{q}} \tag{51}$$

have been used.

Taking into account the rheotropic conditions (39) we can rewrite Eqs. (46) – (49) in the form

$$T^{++} = -2ie^{+q}e^{+q} \tag{52}$$

$$e^{++}\Omega^{--i} = -4ie^{++}e^{+q}\gamma^i_{q\dot{q}}\psi^-_{++\dot{q}} \tag{53}$$

$$T^{+q} = \frac{1}{2}e^{++}\wedge\Omega^{++i}\gamma^i_{q\dot{q}}\psi^-_{++\dot{q}} \tag{54}$$

$$e^{+q}\Omega^{--i}\gamma^i_{q\dot{q}} = 2e^{++}D\psi^-_{++\dot{q}} - 4ie^{+q}e^{+q}\psi^-_{++\dot{q}} \tag{55}$$

Eq. (52) contains the set of the worldline superspace torsion constraints, in particular, the most essential one

$$T_{+q+p}{}^{++} = -4i\delta_{qp} \tag{56}$$

The appearance of the worldvolume superspace torsion constraint as equations of motion of the generalized action for super – p – branes was noted in [1].

Eq. (54) gives the expression for Grassmann torsion two – form. It means that its component with Grassmann indices vanishes

$$T_{+p+r}{}^{+q} = 0 \tag{57}$$

and component with one bosonic index is expressed in terms of $\psi^-_{++\dot{q}}$ and spinor component of Ω^{++i}

$$T_{++\ +p}{}^{+q} = -\frac{1}{2}\Omega^{++i}_{+p}\gamma^i_{q\dot{q}}\psi^-_{++\dot{q}} \tag{58}$$

Note that (58) and, hence, the whole Grassmann torsion two form vanishes on – shell (see Eq. (35) above).

Eq. (55) with taking into account (52) can be decomposed into the following set of relations

$$\Omega^{--i}_{\{+p}\gamma^i_{q\}\dot{q}} = -4i\delta_{qp}\psi^-_{++\dot{q}} \tag{59}$$

$$2D_{+q}\psi^-_{++\dot{q}} = -\Omega^{--i}_{++}\gamma^i_{q\dot{q}} \tag{60}$$

The solution of Eq. (59) has the form [8]

$$\Omega^{--i}_{+q} = -4i\gamma^i_{q\dot{q}}\psi^-_{++\dot{q}} \tag{61}$$

Hence, Eq. (55) determines completely the form Ω^{--i}

$$\Omega^{--i} = -4ie^{+q}\gamma^i_{q\dot{q}}\psi^-_{++\dot{q}} - \frac{2}{D-2}e^{++}D_{+q}\psi^-_{++\dot{q}}\gamma^i_{q\dot{q}} \tag{62}$$

[8] In D=10 the triality relations:

$$\gamma^i_{\{q|\dot{q}}\tilde{\gamma}^j_{\dot{q}|p\}} = \delta^{ij}\delta_{qp}; \quad \tilde{\gamma}^i_{\{\dot{q}|q}\gamma^j_{q|\dot{p}\}} = \delta^{ij}\delta_{\dot{q}\dot{p}}; \quad \gamma^i_{\{q|\dot{q}}\gamma^i_{|p\}\dot{p}} = \delta_{qp}\delta_{\dot{q}\dot{p}}$$

shall be used.

and puts the following restriction on the Grassmann superfield $\psi^{-}_{++\dot{q}}$

$$D_{+q}\psi^{-}_{++\dot{q}} = \frac{1}{D-2}\gamma^{i}_{q\dot{q}}\gamma^{i}_{p\dot{p}}D_{+p}\psi^{-}_{++\dot{p}} \tag{63}$$

It should be note that Eq. (53) as well as the second Peterson – Codazzi equation (14) $D\Omega^{--i} = 0$ are satisfied identically for the form (62), (63). $SO(1,1)$ and $SO(D-2)$ curvatures of worldline superspace are defined by Gauss (15) and Ricci (16) equations and involve the form Ω^{++i}, unrestricted by rheotropic conditions, as the expressions (48), (58) for the Grassmann torsion two – form do.

The restrictions on the form Ω^{++i} follow from the first Peterson – Codazzi equation only

$$\begin{aligned} D\Omega^{++i} = 0 \Longrightarrow e^{+q}D\Omega^{++i}_{+q} + T^{++}\Omega^{++i}_{++} \\ + T^{+q}\Omega^{++i}_{+q} + e^{++}D\Omega^{++i}_{++} = 0 \end{aligned} \tag{64}$$

The independent consequence of Eq. (64) is

$$D_{+\{p}\Omega^{++i}_{+q\}} = 2i\delta_{pq}\Omega^{++i}_{++} \tag{65}$$

where Eqs. (52), (54) are taken into account. Thus,

$$\Omega^{++i} = (e^{+q} - \frac{i}{2(D-2)}e^{++}D_{+q})\Omega^{++i}_{+q}$$

with an arbitrary Grassmann superfield Ω^{++i}_{+q}.

Under the boost transformations (21) this superfield transforms as

$$\delta\Omega^{++i}_{+q} = D_{+q}k^{++i}$$

However, this symmetry can not be used to gauge Ω^{++i}_{+q} to zero, except for the simplest case D=3.

Hence, on mass shell N=1 superparticle can be described by:

1) The covariant Cartan forms

$$\Omega^{++i} = (e^{+q} - \frac{i}{2(D-2)}e^{++}D_{+q})\Omega^{++i}_{+q} \tag{66}$$

$$\Omega^{--i} = -4ie^{+q}\gamma^{i}_{q\dot{q}}\psi^{-}_{++\dot{q}} - \frac{2}{D-2}e^{++}D_{+q}\psi^{-}_{++\dot{q}}\gamma^{i}_{q\dot{q}} \tag{67}$$

where superfield $\psi^{-}_{++\dot{q}}$ is restricted by

$$D_{+q}\psi^{-}_{++\dot{q}} = \frac{1}{D-2}\gamma^{i}_{q\dot{q}}\gamma^{i}_{p\dot{p}}D_{+p}\psi^{-}_{++\dot{p}}. \tag{68}$$

Superfield Ω^{++i}_{+q} (66) satisfying

$$D_{+\{q}\Omega^{++i}_{+p\}} = \delta_{pq}\frac{1}{D-2}D_{+r}\Omega^{++i}_{+r} \tag{69}$$

transforms as follows

$$\delta\Omega^{++i}_{+q} = D_{+q}k^{++i} \tag{70}$$

under the action of (gauge) boost symmetry (21).

2) Worldline supergravity (e^{++}, e^{+q}) satisfying the constraints

$$T^{++} = -2ie^{+q}e^{+q} \tag{71}$$

$$T^{+q} = \frac{1}{2}e^{++}e^{+p}\Omega^{++i}_{+p}\gamma^{i}_{q\dot{q}}\psi^{-}_{++\dot{q}} \tag{72}$$

3) Induced $SO(1,1)$, $SO(D-2)$ connections $\Omega^{(0)}$, Ω^{ij} whose curvatures are determined by Gauss and Ricci equations

$$\mathcal{F} \equiv d\Omega^{(0)} = \frac{1}{2}\Omega^{--i}\Omega^{++i} \tag{73}$$

$$R^{ij} \equiv d\Omega^{ij} + \Omega^{ik}\Omega^{kj} = -\Omega^{--[i}\Omega^{++j]} \tag{74}$$

In the present consideration we have not take into account scalar density superfield ρ^{++}. It can be considered as compensator of $SO(1,1)$ gauge transformations.

2.4 The action independence on the surface.

In this subsection we explain the fact that the generalized action possesses superdiffeomorphism invariance and variation with respect to the surface $\mathcal{M}^1$ does not result in independent equations of motion.

First of all, let us note that the general variation of Lagrangian one – form (including the surface $\mathcal{M}^1$ variation) is

$$\delta\mathcal{L} = (d\mathcal{L})(d,\delta) + d(\mathcal{L}(\delta)) \tag{75}$$

or, in details,

$$\begin{aligned}\delta\mathcal{L} = -d\rho^{++}E^{--}(\delta) + \delta\rho^{++}E^{--}(d) + \rho^{++}E^{--}(\delta)\Omega^{(0)}(d) \\ -\rho^{++}E^{--}(d)\Omega^{(0)}(\delta) - \rho^{++}E^{i}(\delta)\Omega^{--i}(d) + \rho^{++}E^{i}(d)\Omega^{--i}(\delta) \\ - 4i\rho^{++}E^{-\dot{q}}(d)E^{-\dot{q}}(\delta) + d(\rho^{++}E^{--}(\delta))\end{aligned} \tag{76}$$

The last term has not contribution into variation due to the chosen initial conditions. On the surface of the rheotropic relations

$$E^{--} = 0; \qquad E^{i} = 0 \tag{77}$$

the Lagrangian form transforms as follows:

$$\delta\mathcal{L}|_{rh.} = -d\rho^{++}E^{--}(\delta) + \rho^{++}E^{--}(\delta)\Omega^{(0)}(d)$$

$$-\rho^{++}E^{i}(\delta)\Omega^{--i}(d) - 4i\rho^{++}E^{-\dot{q}}(d)E^{-\dot{q}}(\delta) \tag{78}$$

To specify (78), consider the variation of the action (1) under the general coordinate transformations and the change of the surface $\mathcal{M}^1$

$$\delta_{g.c.+surf.}S = \int_{\delta\mathcal{M}^1} \mathcal{L} = \int_{\mathcal{M}^1} (\mathcal{L}(\tau+\delta\tau, \eta^q(\tau)+\delta\eta^q(\tau)) - \mathcal{L}(\tau, \eta^q(\tau))) = \int_{\mathcal{M}^1} \delta_{\tau,\eta}\mathcal{L} \tag{79}$$

where $\delta_{\tau,\eta}\mathcal{L}$ is determined by Eq. (78) with

$$E^{--}(\delta) = \delta\tau\Pi^{\underline{m}}_{\tau}u^{--}_{\underline{m}} + \delta\eta^q\Pi^{\underline{m}}_{q}u^{--}_{\underline{m}} \tag{80}$$

$$E^{i}(\delta) = \delta\tau\Pi^{\underline{m}}_{\tau}u^{i}_{\underline{m}} + \delta\eta^q\Pi^{\underline{m}}_{q}u^{i}_{\underline{m}} \tag{81}$$

$$E^{-\dot{q}}(\delta) = \delta\tau E^{-\dot{q}}_{\tau} + \delta\eta^q E^{-\dot{q}}_{q} \tag{82}$$

The first two variations (80), (81) vanish due to the consequences from the rheotropic relations (77)

$$\Pi^{\underline{m}}_{\tau} = e^{++}_{\tau}u^{--}_{\underline{m}}; \qquad \Pi^{\underline{m}}_{q} = e^{++}_{q}u^{--}_{\underline{m}} \Longrightarrow \Pi^{\underline{m}}_{\tau}u^{--}_{\underline{m}} = 0; \qquad \Pi^{\underline{m}}_{q}u^{--}_{\underline{m}} = 0 \tag{83}$$

and properties of vector harmonics (4).

Further, the variation (82) can not provide a new equation of motion, because it is involved (78) in the combination with $E^{-\dot{q}}(d)$

$$E^{-\dot{q}}(d)E^{-\dot{q}}(\delta) = e^{+p}E^{-\dot{q}}_{+p}(2\delta\tau E^{-\dot{q}}_{\tau} + \delta\eta^q E^{-\dot{q}}_{q}) \tag{84}$$

only. The latter vanishes due to the spinorian rheotropic relation

$$E^{-\dot{q}}_{+p} \equiv D_{+p}\Theta^{\underline{\mu}}v^{-}_{\underline{\mu}\dot{q}} = 0 \tag{85}$$

Hence

$$\delta_{g.c.+surf.}S = 0 \tag{86}$$

holds as a result of the rheotropic conditions only.

All mentioned above make clear the condition of the off – shell superdiffeomorphism invariance in the rheonomic sense, proposed in [41] and considered in [1]. This condition require the vanishing of the external derivative of the Lagrangian form

$$d\mathcal{L} = (d\rho^{++} - \rho^{++}\Omega^{(0)})E^{--} + \rho^{++}E^{i}\Omega^{--i}(d) - i\rho^{++}d\Theta\Gamma^{\underline{m}}d\Theta u^{--}_{\underline{m}} = 0 \tag{87}$$

due to the rheotropic conditions only and the non – dynamical meaning the rheotropic conditions (i.e., the rheotropic conditions shall not result in the proper dynamical equations). In this form the proof of the superdiffeomorphism invariance, being equivalent to the presented above, becomes much more short. Namely, the first two terms in Eq. (87) vanish due to the rheotropic relations (77). The last term (after the complete decomposition onto the supervielbein forms) can be rewritten as follows:

$$-i\rho^{++}e^{+q}(2e^{++}D_{++}\Theta v^{-}_{\dot{q}} + e^{+q}D_{+q}\Theta v^{-}_{\dot{q}})D_{+q}\Theta v^{-}_{\dot{q}}$$

and vanishes due to rheotropic relation (85).

This means the off – shell superdiffeomorphism invariance of N=1 superparticle action in rheonomic sense [41, 1], because, as it have been proved in subsection 2.3, the rheotropic conditions do not lead to equations of motion.

2.5 "On – shell" description of N=1 superparticle in diverse dimensions.

When the proper (dynamical) equation of motion (35) is taken into account

$$\psi^{-}_{++\dot{q}} \equiv D_{++}\Theta^{\underline{\mu}} v^{-}_{\underline{\mu}\dot{q}} = 0 \tag{88}$$

we get from Eq. (62)

$$\Omega^{--i} = 0 \tag{89}$$

Thus, we have proved the dependence of the equation of motion (35).

The curvatures of worldline superspace (see (73), (74)) vanish due to Eq. (89)

$$\mathcal{F} \equiv d\Omega^{(0)} = 0 \tag{90}$$

$$R^{ij} \equiv d\Omega^{ij} + \Omega^{ik}\Omega^{kj} = 0 \tag{91}$$

As a result, the induced connections $\Omega^{(0)}$, Ω^{ij} can be gauged away and, thus, have not physical degrees of freedom.

Eqs. (90), (91) mean that Peterson – Codazzi equation (13) can be solved by

$$\Omega^{++i} = Dl^{++i} \tag{92}$$

Henceforth, superfield l^{++i} transforms additively under the boost symmetry (21)

$$\delta l^{++i} = k^{++i}$$

and can be gauged away

$$l^{++i} = 0$$

In this gauge

$$\Omega^{++i} = 0 \tag{93}$$

and worldline supergravity (e^{++}, e^{+q}) is characterized by the flat constraint

$$T^{++} = -2ie^{+q}e^{+q}; \qquad T^{+q} = 0 \tag{94}$$

To get the general solution of (94) let us consider the exact form of $SO(1,1)$ and $SO(D-2)$ connections being the solution of Eqs. (27), (90) and (91) respectively:

$$\Omega^{(0)} = \frac{d\rho^{++}}{\rho^{++}} \tag{95}$$

$$\Omega^{ij} = dG^{ik}G^{jk}; \qquad G^{ik}G^{jk} = \delta^{ij} \tag{96}$$

In (96) G^{ik} are arbitrary orthogonal matrices.

Further it is convenient to use the spinor representation Ω^{qp} for $SO(D-2)$ connection, which is defined by the relations

$$\Omega^{ij}(d) = \frac{2}{D-2} v_q^{-\underline{\mu}} \gamma^{ij}_{qp} dv^{+}_{\underline{\mu}p} \equiv \frac{2}{D-2} \gamma^{ij}_{qp} \Omega^{qp} \tag{97}$$

$$\Omega^{qp} = \frac{i}{4} (\gamma_{ij})_{qp} \Omega^{ij}; \qquad R^{qp} = \frac{i}{4} (\gamma_{ij})_{qp} R^{ij} = 0;$$
$$\Omega^{qp} = dG^{qr}G^{pr}; \qquad G^{qr}G^{pr} = \delta^{qp} \tag{98}$$

Using Eqs. (95), (98), we can write down the second Eq. (94):

$$T^{+q} \equiv De^{+q} = de^{+q} - \frac{1}{2} e^{+q}\Omega^{(0)} + e^{+p}\Omega^{pq}$$
$$= de^{+q} + \sqrt{\rho^{++}} e^{+q} d(\frac{1}{\sqrt{\rho^{++}}}) + e^{+p} dG^{pk}G^{qk}$$
$$= \sqrt{\rho^{++}} G^{qk} d(\frac{e^{+p}G^{pk}}{\sqrt{\rho^{++}}}) = 0 \tag{99}$$

The later equation can be solved by

$$e^{+p} = \sqrt{\rho^{++}} G^{pq} d\tilde{\eta}^q \tag{100}$$

with arbitrary Grassmann function $\tilde{\eta}^q = \tilde{\eta}^q(\tau, \eta^q)$.

Henceforth, the first equation from (94) becomes

$$T^{++} \equiv De^{++} = de^{++} - e^{++}\Omega^{(0)} = \rho^{++} d(\frac{e^{++}}{\rho^{++}}) = -2i\rho^{++} d\tilde{\eta}^q d\tilde{\eta}^q \tag{101}$$

and has the natural solution

$$e^{++} = \rho^{++}(d\tilde{\tau} - 2id\tilde{\eta}^q\tilde{\eta}^q); \qquad \tilde{\tau} = \tilde{\tau}(\tau, \eta^q) \tag{102}$$

With taking into account (100), (102) and (88), Eqs. (28), (40) acquire the form

$$dX^{\underline{m}} - id\Theta\Gamma^{\underline{m}}\Theta = \frac{1}{2}\rho^{++}(d\tilde{\tau} - 2id\tilde{\eta}^q\tilde{\eta}^q) u^{--\underline{m}} \tag{103}$$

$$d\Theta^{\underline{\mu}} = \sqrt{\rho^{++}} G^{qp} d\tilde{\eta}^p v_q^{-\underline{\mu}} \tag{104}$$

Using the evident relation

$$d\tilde{\eta}^q = d\tau\partial_\tau\tilde{\eta}^q + d\eta^{+p} D_{+p}\tilde{\eta}^q$$

we get from (104)

$$D_{+p}\Theta^{\underline{\mu}} = \sqrt{\rho^{++}} G^{rq} D_{+p}\tilde{\eta}^q v_r^{-\underline{\mu}} \tag{105}$$

$$\partial_\tau\Theta^{\underline{\mu}} = \sqrt{\rho^{++}} G^{qp} \partial_\tau\tilde{\eta}^p v_q^{-\underline{\mu}} \tag{106}$$

Eq. (106) leads to the standard equation of motion for Brink – Schwarz superparticle

$$\Pi_\tau^{\underline{m}}(\Gamma_{\underline{m}})_{\underline{\mu\nu}}\partial_\tau\Theta^{\underline{\mu}}=0 \tag{107}$$

because, due to (103),

$$\Pi_\tau^{\underline{m}}\equiv\partial_\tau X^{\underline{m}}-i\partial_\tau\Theta\Gamma^{\underline{m}}\Theta=\frac{1}{2}\rho^{++}u^{--\underline{m}}(\partial_\tau\tilde{\tau}-2i\partial_\tau\tilde{\eta}^q\tilde{\eta}^q) \tag{108}$$

and

$$u_{\underline{m}}^{--}\Gamma_{\underline{\mu\nu}}^{\underline{m}}v_q^{-\underline{\nu}}=2v_{\underline{\mu}\dot{q}}^{-}v_{\underline{\nu}\dot{q}}^{-}v_q^{-\underline{\nu}}=0$$

Using the general coordinate transformations in worldline superspace we can fix the gauge $\tilde{\tau}=\tau$, $\tilde{\eta}^q=\eta^q$, which is conserved by superconformal transformations. In this gauge

$$e^{+q}=\sqrt{\rho^{++}}d\eta^q;\qquad e^{++}=\rho^{++}(d\tau-2id\eta^q\eta^q)$$

From the other hand, using Eqs. (90) and (95), we can fix the gauge $\Omega^{(0)}=0$ by imposing $\rho^{++}=const$, and all the Cartan forms vanish

$$\Omega^{\underline{ab}}=0$$

In this gauge harmonics u and v are the constant vectors and spinors constrained by (4), (5) respectively

$$du=0;\qquad dv=0$$

R.h.s. of Eqs. (28), (40) becomes very simple

$$dX^{\underline{m}}-id\Theta\Gamma^{\underline{m}}\Theta=\frac{1}{2}(d\tau-2id\eta^q\eta^q)u^{--\underline{m}}$$

$$d\Theta^{\underline{\mu}}=d\eta^q v_q^{-\underline{\mu}}$$

and can be used to extract the solution of mass shell constraints

$$\dot{x}^{\underline{m}}=\frac{1}{2}u^{--\underline{m}}=\frac{1}{2(D-2)}v_q^{-}\Gamma^{\underline{m}}v_q^{-}\Longrightarrow(\dot{x}^{\underline{m}})^2=0$$

as well as equations of motion (in the form, possessing only the global woldline supersymmetry [3])

$$\ddot{x}=0;\qquad \dot{\theta}^{\underline{\mu}}=0$$

for leading components of the superfields X and Θ,

$$x^{\underline{m}}=X^{\underline{m}}(\tau,\eta=0);\qquad \theta^{\underline{\mu}}=\Theta^{\underline{\mu}}|_{\eta=0}$$

and expressions for their higher components in terms of x, θ and constant twistor – like variables v_q^-

$$D_{+q}X^{\underline{m}}|_{\eta=0}=-2i\eta^q u^{--\underline{m}}+iv_q^{-}\Gamma^{\underline{m}}\theta$$

$$D_{+q}\Theta^{\underline{\mu}}|_{\eta=0}=v_q^{-\underline{\mu}}$$

etc.

Hence, we have proved the classical equivalence of N=1 superparticle description in the framework of the generalized action principle approach with standard one.

2.6 Action in analytical basis.

In the analytical coordinate basis $\mathcal{Z}^{\underline{\mathcal{A}}} = (\tilde{Z}^{\underline{A}}, v_{\underline{\mu}}^{\underline{\alpha}}) = (X^{++}, X^{--}, \tilde{X}^i, \Theta^{\underline{\alpha}}, v_{\underline{\mu}}^{\underline{\alpha}})$

$$X^{\underline{a}} \equiv X^{\underline{m}} u_{\underline{m}}^{\underline{a}} = (X^{++}, X^{--}, X^i) = (X^{\underline{m}} u_{\underline{m}}^{++}, X^{\underline{m}} u_{\underline{m}}^{--}, X^{\underline{m}} u_{\underline{m}}^{i})$$

$$\Theta^{\underline{\alpha}} \equiv \Theta^{\underline{\mu}} v_{\underline{\mu}}^{\underline{\alpha}} = (\Theta^{+q}, \Theta^{-\dot{q}}) = (\Theta^{\underline{\mu}} v_{\underline{\mu}}^{+q}, \Theta^{\underline{\mu}} v_{\underline{\mu}}^{-\dot{q}})$$

$$\tilde{X}^i = X^i + \frac{i}{4} \Theta_q^+ \gamma_{q\dot{q}}^i \Theta_{\dot{q}}^-$$

of a target superspace the action (1) acquires the form

$$S = \int_{\mathcal{M}^1} \rho^{++} (DX^{--} - 2iD\Theta_{\dot{q}}^- \Theta_{\dot{q}}^- - \tilde{X}^i \Omega^{--i}) \tag{109}$$

where [9]

$$DX^{--} \equiv dX^{--} + X^{--} \Omega^{(0)}(d)$$
$$D\Theta^{-\dot{q}} \equiv d\Theta^{-\dot{q}} + \frac{1}{2} \Theta^{-\dot{q}} \Omega^{(0)} - \frac{1}{4} \Omega^{ij} \tilde{\gamma}_{\dot{q}\dot{p}}^{ij} \Theta^{-\dot{p}}$$
$$\tilde{X}^i = X^i + \frac{i}{4} \Theta_q^+ \gamma_{q\dot{q}}^i \Theta_{\dot{q}}^- \tag{110}$$

The point is important that the functional (109) does not involve the variables Θ^{+q} and X^{++}, which transform additively under (irreducible superfield) κ – symmetry (22) and b – symmetry (23) respectively (i.e. are the compensators of these symmetries).

Such type effect was found by E.Sokatchev [43] (in superparticle action written in Hamiltonian form with use of vector Lorentz harmonic variables).

This observation seems to be important for further attempts to find a way for superparticle covariant quantization in the framework of the generalized action approach.

3 Coupling to Maxwell background field.

The generalized action describing superparticle interacting with Abelian background has the form

$$S = \int_{\mathcal{M}^1} (\rho^{++} E^{--} + \mathcal{A}) \tag{111}$$

where, as in (1), $\mathcal{M}^1 = \{(\tau, \eta^q) : \eta^q = \eta^q(\tau)\}$ is an arbitrary bosonic line in worldline superspace. The superfield ρ^{++} and the form E^{--} are defined in section 2.1 and

$$\mathcal{A} = dZ^{\underline{M}} \mathcal{A}_{\underline{M}}(X^{\underline{m}}(\tau, \eta^{+q}), \Theta^{\underline{\mu}}(\tau, \eta^{+q})); \qquad Z^{\underline{M}} = \{X^{\underline{m}}, \Theta^{\underline{\mu}}\} \tag{112}$$

[9]It shall be noted that the shift of the transverse components similar to the (110) one plays an important role in the investigation performed in [53], where the linearized version of the formalism [19] was developed and applied for a wide class of supersymmetric extended objects, including D=11 super – five brane.

To obtain equations of motion we have to vary the action (111) with taking into account the fact that a differential form variation is similar to the Lie derivative operation, i.e. (see subsection 3.1 below)

$$\delta \mathcal{A} = (d\mathcal{A})(d, \delta) + d(\mathcal{A}(\delta)) \tag{113}$$

Under assumption about trivial initial conditions, the contribution from the last term of (113) vanishes and

$$\int_{\mathcal{M}^1} \delta \mathcal{A} \equiv \int_{\mathcal{M}^1} F(d, \delta) = \int_{\mathcal{M}^1} E^{\underline{B}}(d) E^{\underline{A}}(\delta) F_{\underline{AB}} \tag{114}$$

where $F_{\underline{AB}}$ is the strength tensor superfield defined by

$$d\mathcal{A} = \frac{1}{2} E^{\underline{B}} E^{\underline{A}} F_{\underline{AB}} \equiv F; \qquad F_{\underline{AB}} = -(-)^{\underline{AB}} F_{\underline{BA}} \tag{115}$$

In Eqs. (114), (115) $E^{\underline{A}}$ can be regarded as a set of basic forms $\Pi^{\underline{M}} = (\Pi^{\underline{m}}, d\Theta^{\underline{\mu}})$ (2), or as the frame $E^{\underline{A}}$ from Eq. (3). In the first case the field strength components $F_{\underline{AB}} = F_{\underline{AB}}(X, \Theta)$ depend on the (super)field $X^{\underline{m}}$ and $\Theta^{\underline{\mu}}$ only, while in the second case the field strength components $F_{\underline{AB}}$ do depend bilinearly on the u and v matrices (4), (5)

$$F_{\underline{\alpha\beta}} = v^{\underline{\mu}}_{\underline{\alpha}} v^{\underline{\nu}}_{\underline{\mu}} F_{\underline{\mu\nu}}(X, \Theta) \equiv v_{\underline{\alpha}} F v_{\underline{\beta}}$$

$$F_{\underline{\alpha a}} = v^{\underline{\mu}}_{\underline{\alpha}} u^{\underline{m}}_{\underline{a}} F_{\underline{\mu m}} = v_{\underline{\alpha}} F u_{\underline{a}}$$

$$F_{\underline{ab}} = u^{\underline{m}}_{\underline{a}} u^{\underline{n}}_{\underline{b}} F_{\underline{mn}} = u_{\underline{a}} F u_{\underline{b}} \tag{116}$$

3.1 Restrictions on background field.

Let us begin from a consideration of the complete set of equations of motion following from the action (111) with an arbitrary Abelian one – form

$$\frac{\delta S}{\delta \rho^{++}} = 0 \Longrightarrow E^{--}(d) = 0 \tag{117}$$

$$\frac{\delta S}{\Omega^{--i}(\delta)} \equiv u^{i}_{\underline{m}} \frac{\delta S}{\delta u^{--}_{\underline{m}}} = 0 \Longrightarrow \rho^{++} E^{i}(d) = 0 \tag{118}$$

$$\frac{\delta S}{E^{-\dot{q}}(\delta)} \equiv v^{+\underline{\mu}}_{\dot{q}} \frac{\delta S}{\delta \Theta^{\underline{\mu}}} = 0 \Longrightarrow$$

$$E^{-\dot{p}}(v^{+}_{\dot{p}} F v^{+}_{\dot{q}} - 4i\delta_{\dot{p}\dot{q}} \rho^{++}) + E^{+q}(v^{-}_{q} F v^{+}_{\dot{q}}) + E^{++}(\frac{1}{2} u^{--} F v^{+}_{\dot{q}}) = 0 \tag{119}$$

$$\frac{\delta S}{E^{--}(\delta)} \equiv u^{\underline{m}++} \frac{\delta S}{\delta X^{\underline{m}}} = 0 \Longrightarrow$$

$$D\rho^{++} = \frac{1}{4} E^{++}(d) u^{++} F u^{--} + \frac{1}{2} E^{+q}(d) u^{++} F v^{-}_{q} + \frac{1}{2} E^{-\dot{q}}(d) u^{++} F v^{+}_{q}$$

$$\frac{\delta S}{E^{i}(\delta)} \equiv u^{\underline{m}i} \frac{\delta S}{\delta X^{\underline{m}}} = 0 \Longrightarrow$$

$$\rho^{++}\Omega^{--i}(d) = -\frac{1}{2}E^{++}(d)u^i F u^{--} + E^{+q}(d)u^i F v^-_q + E^{-\dot{q}}u^i F v^+_{\dot{q}} \tag{120}$$

$$\frac{\delta S}{E^{++}(\delta)} \equiv u^{\underline{m}--}\frac{\delta S}{\delta X^{\underline{m}}} = 0 \Longrightarrow u^{--}F v^-_q E^{+q}(d) + u^{--}F v^+_{\dot{q}} E^{-\dot{q}} = 0 \tag{121}$$

$$\frac{\delta S}{E^{+q}(\delta)} \equiv v^{-\underline{\mu}}\frac{\delta S}{\delta \Theta^{\underline{\mu}}} = 0 \Longrightarrow v^-_q F v^-_p E^{+p}(d) + v^-_q F v^+_{\dot{q}} E^{-\dot{q}} = 0 \tag{122}$$

where the later equations are written with taking into account Eqs. (117), (118).

Equation

$$\frac{\delta S}{\Omega^{(0)}(\delta)} \equiv -u^{--}_{\underline{m}}\frac{\delta S}{\delta u^{--}_{\underline{m}}} = 0 \tag{123}$$

is satisfied identically after Eq. (117) is taken into account. This is Noether identity corresponding to $SO(1,1)$ gauge symmetry of the generalized action. The Noether identities reflecting $SO(D-2)$ and K_{D-2} (boost) symmetry of the generalized action are

$$\frac{\delta S}{\Omega^{ij}(\delta)} \equiv 0 \Longleftrightarrow \ 0 = 0 \tag{124}$$

$$\frac{\delta S}{\Omega^{++i}(\delta)} \equiv 0 \Longleftrightarrow \ 0 = 0 \tag{125}$$

As in the free superparticle case, they appear due to the absence of moving frame vectors $u^{++}_{\underline{m}}$ and $u^i_{\underline{m}}$ in the generalized action (111).

Eq. (122) and (121) are satisfied identically in the absence of background field. These are the Noether identities reflecting worldline supersymmetry (realized nonlinearly in the form of irreducible superfield κ – symmetry) and reparametrization symmetry (realized as b – symmetry) of the generalized action (1).

The supposition about continuous zero field limit requires the existence of such symmetries for nonvanishing background (see Ref. [44] and Refs. therein). Following these conjecture we shall require the background field to be restricted by constraints which shall turn Eqs. (122), (121) into identities after Eqs. (117) – (120) are taken into account. So Eqs. (122), (121) shall be treated as equations for background, which shall be satisfied for any values of worldline superfields, restricted by Eqs. (117) – (120).

Because the Abelian connection form coefficients (112) are supposed to be independent on harmonic superfields u and v, the general solution of Eq. (122) with respect to the field strength is

$$F_{\underline{\mu\nu}}(X,\Theta) = 0 \tag{126}$$

Indeed, this relation results in

$$v^-_q F v^-_p = 0 \tag{127}$$

as well as in

$$v^-_q F v^+_{\dot{q}} = 0; \qquad v^+_{\dot{q}} F v^+_{\dot{p}} = 0 \tag{128}$$

To prove that (126) leads also to vanishing of $u^{--}Fv_q^-$ we shall take into account the nontrivial parts of the Bianchi identities for the Abelian field strength

$$dF = 0 \tag{129}$$

which acquire the form

$$\Gamma^{\underline{m}}_{\{\underline{\nu}\underline{\rho}|}F_{\underline{m}|\underline{\mu}\}} = 0 \tag{130}$$

$$\nabla_{\{\underline{\mu}}\, F_{\underline{\nu}\}\underline{n}} - 2i\Gamma^{\underline{m}}_{\underline{\mu}\underline{\nu}}F_{\underline{mn}} = 0 \tag{131}$$

with $\nabla_{\underline{\mu}}$ being a target superspace covariant derivative.

Contracting Eq. (130) with harmonics $v_q^{-\underline{\mu}}v_p^{-\underline{\nu}}v_r^{-\underline{\rho}}$ we get

$$0 = v_{\{q}^{-\underline{\mu}}v_p^{-\underline{\nu}}v_{r\}}^{-\underline{\rho}}\Gamma^{\underline{m}}_{\underline{\mu}\underline{\nu}}F_{\underline{\rho m}} = \delta_{\{qp}v_{r\}}^{-}Fu^{--} \Longrightarrow v_q^-Fu^{--} = 0 \tag{132}$$

For the further consideration the following results of investigation of the Bianchi identities (130), (131) will be useful. The general solution of Eq. (130) is (see [44] and Refs. therein)

$$F_{\underline{m}\underline{\mu}} = (\Gamma_{\underline{m}})_{\underline{\mu}\underline{\nu}}W^{\underline{\nu}} \tag{133}$$

with a Grassmann superfield $W^{\underline{\nu}}$. Then Eq. (131) acquires the form

$$(\Gamma_{\underline{m}})_{\{\underline{\mu}|\underline{\rho}}\, \nabla_{|\underline{\nu}\}}\, W^{\underline{\rho}} - i\Gamma^{\underline{n}}_{\underline{\mu}\underline{\nu}}F_{\underline{nm}} = 0 \tag{134}$$

In the most complicated D=10 case, Eq. (134) expresses the bosonic component $F_{\underline{mn}}$ in terms of spinor target – space derivative of superfield W

$$F_{\underline{mn}} = \frac{i}{2(D-2)}\, \nabla_{\underline{\mu}}\, W^{\underline{\nu}}(\Gamma_{\underline{mn}})_{\underline{\nu}}^{\underline{\mu}} \tag{135}$$

and, besides, puts the following restrictions on W superfield

$$\nabla_{\underline{\mu}}W^{\underline{\mu}} = 0$$

$$\nabla_{\underline{\mu}}\, W^{\underline{\nu}}(\Gamma_{\underline{m}_1\ldots\underline{m}_4})_{\underline{\nu}}^{\underline{\mu}} = 0 \tag{136}$$

As a result of (136), Eq. (135) can be solved with respect to $\nabla_{\underline{\mu}}W^{\underline{\nu}}$ by

$$\nabla_{\underline{\mu}}\, W^{\underline{\nu}} = \frac{i}{2}F^{\underline{mn}}(\Gamma_{\underline{mn}})_{\underline{\mu}}^{\underline{\nu}} \tag{137}$$

To justify that Eq. (121) with background, defined by the constraint (126)

$$u^{--}Fv_{\dot{q}}^{+}E^{-\dot{q}} = 0, \tag{138}$$

is also satisfied identically, Eq. (119) with such background

$$4i\rho^{++}E^{-\dot{q}}(d) = -\frac{1}{2}v_{\dot{q}}^{+}Fu^{--}E^{++}(d) \tag{139}$$

shall be used

$$u^{--}Fv^{+}_{\dot q}E^{-\dot q}(d) = -\frac{i}{8\rho^{++}}u^{--}Fv^{+}_{\dot q}u^{--}Fv^{+}_{\dot q}E^{++}(d) \equiv 0$$

Hence, for the background satisfying the standard constraint (126), all the symmetries of free superparticle action are conserved.

Indeed, the variation of the action (111) involving backgroung field (112) satisfying (126) acquires the form

$$\begin{aligned}
\delta S|_{F_{\underline{\mu\nu}}=0} = \int_{\mathcal{M}^1} & (\delta\rho^{++} - \rho^{++}\Omega^{(0)}(\delta) + E^{++}(\delta)u^{--}Fu^{++} \\
& -E^{i}(\delta)u^{i}Fu^{++} + E^{+q}(\delta)v^{-}_{q}Fu^{++})E^{--}(d) \\
& + \int_{\mathcal{M}^1} (\rho^{++}\Omega^{--i}(\delta) - E^{\underline{a}}(\delta)u_{\underline{a}}Fu^{i} - E^{\underline{\alpha}}(\delta)v_{\underline{\alpha}}Fu^{i})E^{i}(d) \\
& + \int_{\mathcal{M}^1} (E^{-\dot q}(\delta) + \frac{i}{\rho^{++}}E^{++}(\delta)u^{--}Fv^{+}_{\dot q})(-4i\rho^{++}E^{-\dot q} + v^{+}_{\dot q}Fu^{--}E^{++}(d)) \\
& - \int_{\mathcal{M}^1} E^{i}(\delta)(\rho^{++}\Omega^{--i}(d) + \frac{1}{2}u^{i}Fu^{--}E^{++}(d) - u^{i}Fv^{-}_{q}E^{+q} - u^{i}Fv^{+}_{\dot q}E^{-\dot q}) \\
& - \int_{\mathcal{M}^1} E^{--}(\delta)(D\rho^{++} - \frac{1}{4}u^{++}Fu^{--}E^{++}(d) + \frac{1}{2}u^{++}Fv^{-}_{q}E^{--}(d))
\end{aligned} \tag{140}$$

and contains essentially only the same independent variations $\delta\rho^{++}$, $\Omega^{--i}(\delta)$, $E^{-\dot q}(\delta)$, $E^{--}(\delta)$ as a variation of free superparticle action (18) does (for brevity the explicit expressions for the field strength components in terms of $W^{\underline{\mu}}$ superfield are not specified in (140)).

3.2 Extrinsic geometry of N=1 superparticle in Abelian background.

Equations of motion derived from (140) by a variation over ρ^{++}, $\Omega^{--i}(\delta)$, $E^{--i}(\delta)$, $E^{i}(\delta)$ and $E^{-\dot q}(\delta) = \delta\Theta v^{-}_{\dot q}$ variables have the following form:

$$E^{--} = 0; \qquad E^{i} = 0 \tag{141}$$

$$E^{-\dot q} = \frac{1}{8i\rho^{++}}E^{++}u^{--}Fv^{+}_{\dot q} = \frac{1}{4i\rho^{++}}e^{++}W^{\underline{\nu}}v^{-}_{\underline{\nu}\dot q} \tag{142}$$

$$\begin{aligned}
\Omega^{--i}(d) &= \frac{1}{\rho^{++}}E^{\underline{B}}F_{\underline{Bm}}u^{i\underline{m}} \\
&= \frac{1}{\rho^{++}}e^{+q}\gamma^{i}_{q\dot q}Wv^{-}_{\dot q} + \frac{i}{16}e^{++}(\frac{1}{\rho^{++}}W^{-}_{q}\bigtriangledown Wv^{+}_{\dot q}\gamma^{i}_{q\dot q} + \frac{4}{(\rho^{++})^2}Wv^{+}_{q}\gamma^{i}_{q\dot q}Wv^{-}_{\dot q})
\end{aligned} \tag{143}$$

$$D\rho^{++} = -\frac{1}{2}E^{\underline{B}}F_{\underline{Bm}}u^{++\underline{m}} = e^{+q}W^{\underline{\mu}}v^{+}_{\underline{\mu}q} - \frac{i}{2(D-2)}e^{++}v^{+\underline{\nu}}_{\dot q}\bigtriangledown_{\underline{\nu}}W^{\underline{\mu}}v^{-}_{\underline{\mu}\dot q} \tag{144}$$

where the conventional rheotropic conditions

$$\Omega^{(0)}(D) = 0; \qquad \Omega^{ij}(D) = 0$$

$$E^{++}(d) = e^{++}(d); \qquad E^{+q}(d) = e^{+q}(d) \tag{145}$$

are chosen in the same form as for the free superparticle case (see subsection 2.2).

It is easy to see that for superparticle interacting with Abelian gauge background the master equations (28), (39)

$$\Pi^{\underline{m}} = \frac{1}{2} e^{++} u^{--\underline{m}} \tag{146}$$

$$d\Theta^{\underline{\mu}} = e^{+q} v_q^{-\underline{\mu}} + e^{++} \psi^{-}_{++\dot{q}} v_{\dot{q}}^{+\underline{\mu}} \tag{147}$$

describing superparticle theory off mass – shell, are satisfied.

Eqs. (142), (147) identify $\psi^{-}_{++\dot{q}}$ superfield with the main background field strength $W^{\underline{\mu}}$ contracted with the harmonic $v^{-}_{\underline{\mu}\dot{q}}$

$$\psi^{-}_{++\dot{q}} = \frac{1}{4i\rho^{++}} W^{\underline{\mu}} v^{-}_{\underline{\mu}\dot{q}} \tag{148}$$

Taking it into account we can use Eqs. (66) – (74) for the description of superparticle motion in Abelian background.

The only problem which can be pointed by the careful reader is related to the Eq. (68) which, after substitution (148) acquire the form

$$D_{+q}(\frac{1}{\rho^{++}} W^{\underline{\mu}} v^{-}_{\underline{\mu}\dot{q}}) = \gamma^{i}_{q\dot{q}} \frac{1}{D-2} \gamma^{i}_{p\dot{p}} D_{+p}(\frac{1}{\rho^{++}} W^{\underline{\mu}} v^{-}_{\underline{\mu}\dot{p}}) \tag{149}$$

and seems to be able to produce an additional restriction on the background superfield $W^{\underline{\mu}}(X,\Theta)$. However, it can be proved (see Appendix) that Eq. (149) is satisfied identically, when Eqs. (144), (137), (65), (43), (148) are taken into account.

Hence, superparticle motion in the Abelian Yang – Mills background, satisfying the standard constraints (126), is described by

1. Worldline supergravity (e^{++}, e^{+q}) satisfying the torsion constraint

$$T^{++} = -2ie^{+q}e^{+q}; \qquad T^{+q} = \frac{1}{8i\rho^{++}} e^{++} \Omega^{++i} \gamma^{i}_{q\dot{q}} W^{\underline{\mu}} v^{-}_{\underline{\mu}\dot{q}}$$

2. Covariant one – form

$$\Omega^{--i} = \frac{1}{\rho^{++}} e^{+q} \gamma^{i}_{q\dot{q}} W v^{-}_{\dot{q}} + \frac{i}{16} e^{++} (\frac{1}{\rho^{++}} W^{-}_{q} \bigtriangledown W v^{+}_{\dot{q}} \gamma^{i}_{q\dot{q}} + \frac{4}{(\rho^{++})^2} W v^{+}_{q} \gamma^{i}_{q\dot{q}} W v^{-}_{\dot{q}})$$

3. Covariant one – form Ω^{++i} (66)

$$\Omega^{++i} = (e^{+q} - \frac{i}{2(D-2)} e^{++} D_{+q}) \Omega^{++i}_{+q}$$

which is restricted only by the Peterson – Codazzi equations

$$D\Omega^{++i} = 0 \Longrightarrow \ D_{+\{q}\Omega_{+p\}} = 2i\delta_{qp}\Omega^{++i}_{++}$$

and is defined up to the boost symmetry transformations (21)

$$\delta\Omega^{++i}_{+p} = D_{+p}k^{++i}$$

4. $SO(1,1)$ (spin) connection one – form $\Omega^{(0)}$ expressed in terms of $W^{\underline{\mu}}$, harmonics and $SO(1,1)$ compensator ρ^{++} by Eq. (144)

$$\Omega^{(0)}(d) = \frac{1}{\rho^{++}}(d\rho^{++} + e^{+q}Wv^{+}_{q} + \frac{i}{2(D-2)}e^{++}v^{-}_{q} \bigtriangledown Wv^{+}_{q})$$

   $SO(1,1)$ curvature of these forms is determined by the Gauss equation

$$\mathcal{F} \equiv d\Omega^{(0)} = \frac{1}{2}\Omega^{--i}\Omega^{++i}$$

   with the forms $\Omega^{\pm\pm i}$ specified above

5. $SO(D-2)$ connection one – form Ω^{ij} restricted by the Ricci equation [10]

$$R^{ij} \equiv d\Omega^{ij} + \Omega^{ik}\Omega^{kj} = -\Omega^{--[i}\Omega^{++j]}$$

The only fact, which should be stressed, is that all equations describing the superparticle motion in Abelian background can be regarded as ones following from the Peterson – Codazzi, Gauss and Ricci equations for the forms Ω^{--i}, Ω^{++i}, $\Omega^{(0)}$ and Ω^{ij} specified above. Furthermore, the latter equations can be collected into one zero curvature representation, which is given by the Maurer – Cartan equations (12). This reflects the fact that (super)particle equations in external field can be always solved, at least formally.

4 Conclusion.

In conclusion, we have constructed the generalized action principle for N=1 superparticle in flat target superspace of bosonic dimensions D=3,4,6 and 10. We have developed the general framework of the doubly supersymmetric geometrical approach for superparticle and have investigated the general aspects of superparticle interaction with Abelian Yang – Mills background. It was shown that the interaction with superparticle put the restrictions

[10] In the case of extended worldline supersymmetry the higher components (of dimension $> +3/2$) of these equations are dependent that can be seen by studying "identities for identities" being the integrability conditions for the Peterson – Codazzi, Gauss and Ricci equations (see ref. [51] and Refs. therein). For the case of n=1 worldline supersymmetry, realizing in the D=3 superparticle theory, one irreducible part of the dimension +2 identity is independent (see Appendix C of ref. [19]).

on the Abelian background superfield being the standard constraints of supersymmetric gauge theory (see [44] and Refs. therein).

We have demonstrated that transition to analytical basis can be done in such a way that the dependence on the coordinates Θ^{+q} and X^{++} (having the properties of compensators with respect to irreducible κ – symmetry and b – symmetry) disappears from the generalized action.

The presented results can be regarded as a basis for the future attempts to find a way for covariant quantization of superparticles and superstrings in the frame of the generalized action concept, as well as a toy model for studying a general feature of geometrical approach for supersymmetric objects interacting with natural (supergravity and super – Yang – Mills) background.

The most straightforward development of the results of this work consists in the construction of the generalized action and geometrical approach for N=2 superparticles [11] as well as for N=1 null – super – p – branes (tensionless super – p – branes). These problems are under investigations now.

At the same time, N=IIA superparticle is the simplest member of N=2 supersymmetric solitons family in D=10 [52], for higher members of which any actions are unknown up to now.

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Appendix

Here we will prove that the restriction

$$D_{+q}\psi_{++\dot{q}}{}^{-} = \gamma^i_{q\dot{q}}\frac{1}{D-2}\gamma^i_{p\dot{p}}D_{+p}\psi_{++\dot{p}}{}^{-} \qquad (150)$$

[11] We shall stress that the action for N=IIA, IIB D=10 superparticle does not exist in the standard (STVZ – like) doubly supersymmetric approach.

is satisfied identically for

$$\psi_{++\dot{q}}{}^{-} = -\frac{i}{4\rho^{++}} W^{\underline{\mu}} v^{-}_{\underline{\mu}\dot{q}} \tag{151}$$

where $W^{\underline{\mu}}$ is the Abelian Yang – Mills field strength (133), satisfying

$$\nabla_{\underline{\mu}} W^{\underline{\mu}} = 0 \tag{152}$$

$$\nabla_{\underline{\mu}} W^{\underline{\nu}} = \frac{i}{2} F^{\underline{mn}} (\Gamma_{\underline{mn}})_{\underline{\mu}}^{\underline{\nu}} \tag{153}$$

due to Bianchi identities and Eq. (126). Eqs. (152), (153) involve, in particular, equation of motion.

To prove Eq. (150) for superfield (151) in D=10 case, it is enough to prove that

$$4i(\gamma^{j_1 j_2 j_3})_{q\dot{q}} D_{+q} \psi_{++\dot{q}}{}^{-} \equiv (\gamma^{j_1 j_2 j_3})_{q\dot{q}} D_{+q} (\frac{1}{\rho^{++}} W^{\underline{\mu}} v^{-}_{\underline{\mu}\dot{q}}) = 0 \tag{154}$$

Taking into account Eqs. (143), (144), (71), (72), (43), (37), (32), (61) we can write

$$\begin{aligned} 4i D_{+q} \psi_{++\dot{q}}{}^{-} &= D_{+q} (\frac{1}{\rho^{++}} W^{\underline{\mu}} v^{-}_{\underline{\mu}\dot{q}}) = -\frac{1}{(\rho^{++})^2} D_{+q} \rho^{++} W^{\underline{\mu}} v^{-}_{\underline{\mu}\dot{q}} \\ &+ \frac{1}{\rho^{++}} D_{+q} \mathcal{Z}^{\underline{M}} \nabla_{\underline{M}} W^{\underline{\mu}} v^{-}_{\underline{\mu}\dot{q}} - \frac{1}{\rho^{++}} W^{\underline{\mu}} D_{+q} v^{-}_{\underline{\mu}\dot{q}} \\ &= \frac{2}{(\rho^{++})^2} W^{\underline{\mu}} v^{+}_{\underline{\mu}q} W^{\underline{\mu}} v^{-}_{\underline{\mu}\dot{q}} - \frac{1}{(\rho^{++})^2} \gamma^{i}_{q\dot{q}} \gamma^{i}_{p\dot{p}} W^{\underline{\mu}} v^{+}_{\underline{\mu}p} W^{\underline{\mu}} v^{-}_{\underline{\mu}\dot{q}} \\ &+ \frac{1}{(\rho^{++})^2} v_q^{-\underline{\mu}} v^{-}_{\underline{\nu}\dot{q}} \nabla_{\underline{\mu}} W^{\underline{\nu}} \end{aligned} \tag{155}$$

Contracting (155) with $(\gamma^{klm})_{q\dot{q}}$, one could find, that the contributions from the first two terms in r.h.s. of Eq. (155) into Eq. (154) cancel one another due to identity

$$(\tilde{\gamma}^i \gamma^{klm} \tilde{\gamma}^i)_{q\dot{q}} = 2(\gamma^{klm})_{q\dot{q}}$$

which holds for the eight – dimensional σ – matrices $\gamma^{i}_{q\dot{q}}$.

The contribution from the third term becomes proportional to

$$(\gamma^{ij'j})_{q\dot{q}} v_q^{-\underline{\mu}} (\Gamma_{\underline{mn}})_{\underline{\mu}}^{\underline{\nu}} v^{-}_{\underline{\nu}\dot{q}}$$

when Eq. (153) is taken into account. To prove that it vanishes, let us perform the following algebraic manipulations using the condition of Γ – matrices invariance (6)

$$\begin{aligned} v_{\underline{\alpha}}^{\underline{\mu}} (\Gamma_{\underline{m}} \Gamma_{\underline{n}})_{\underline{\mu}}^{\underline{\nu}} v_{\underline{\nu}}^{\underline{\beta}} &= \delta_{\underline{\alpha}}^{\underline{\beta}} + v_{\underline{\alpha}}^{\underline{\mu}} (\Gamma_{\underline{mn}})_{\underline{\mu}}^{\underline{\nu}} v_{\underline{\nu}}^{\underline{\beta}} = v_{\underline{\alpha}}^{\underline{\mu}} (\Gamma_{\underline{m}})_{\underline{\mu\nu}} v_{\underline{\gamma}}^{\underline{\rho}} v_{\underline{\rho}}^{\underline{\gamma}} (\Gamma_{\underline{n}})^{\underline{\mu\nu}} v_{\underline{\nu}}^{\underline{\beta}} \\ &= u_{\underline{m}}^{\underline{a}} (\Gamma_{\underline{m}})_{\underline{\alpha\gamma}} (\Gamma_{\underline{b}})^{\underline{\gamma\beta}} u_{\underline{n}}^{\underline{b}} = u_{\underline{m}}^{\underline{a}} u_{\underline{n}}^{\underline{b}} (\Gamma_{\underline{a}} \Gamma_{\underline{b}})_{\underline{\alpha}}^{\underline{\beta}} \end{aligned} \tag{156}$$

Taking in Eq. (156) $\underline{\alpha} = ({}^{-}_{q})$, $\underline{\beta} = (-\dot{q})$ and using the explicit representation for Γ – matrices (see [23, 26]), we get

$$v_q^{-\underline{\mu}} (\Gamma_{\underline{mn}})_{\underline{\mu}}^{\underline{\nu}} v^{-}_{\underline{\nu}\dot{q}} = 2 u^{--}_{[\underline{m}} u^{i}_{\underline{n}]} \gamma^{i}_{q\dot{q}}$$

and, hence

$$(\gamma^{j_1 j_2 j_3})_{q\dot{q}} v_q^{-\underline{\mu}} (\Gamma_{\underline{mn}})_{\underline{\mu}}^{\underline{\nu}} v^{-}_{\underline{\nu}\dot{q}} = 2 u^{--}_{[\underline{m}} u^{i}_{\underline{n}]} \; Tr\; (\tilde{\gamma}^i \gamma^{j_1 j_2 j_3}) = 0$$

So, we have proved that superfield (151) satisfies the relation (150).

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